Es. 1

(i)

$X \sim B(400, \frac{1}{5})$ $\quad E[X] = 400 \cdot 0.2 = 80$ $\quad Var(X) = \frac{4}{25}$

$$\frac{X - 400 \cdot 0.2}{20\sqrt{16}} = \frac{X-80}{8} \sim N(0,1)$$

$$P\{X \geq 90\} = P\left\{\frac{X-80}{8} \geq \frac{90-80}{8}\right\} \sim 1 - \Phi(1.25) = 0.106$$

(ii)

$Y \sim N(0,1)$ $\quad P\{Y > d\} = 0.05$ se $d = q_{0.95} = 1.65$

$$\frac{90 - 400p}{20\sqrt{p(1-p)}} \sim 1.65 \qquad p = 0.19 \longrightarrow 1.78$$

Es. 2

(i)

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \iff \int_{-1}^{0} -x\,dx + \int_{0}^{1} ax+b = -\left[\frac{x^2}{2}\right]_{-1}^{0} + a\left[\frac{x^2}{2}\right]_0^1 + \left[bx\right]_0^1 =$$

$$= \frac{1}{2} + \frac{a}{2} + b = 1 \iff \frac{1+a+2b}{2} = 1 \iff 1+a+2b = 2 \iff$$

$$\iff a + 2b = 1 \qquad a = 1 - 2b$$

(ii)

$$E[X] = \int_{-1}^{0} -x^2\,dx + \int_0^1 ax^2 + bx\,dx = -\left[\frac{x^3}{3}\right]_{-1}^{0} + a\left[\frac{x^3}{3}\right]_0^1 + b\left[\frac{x^2}{2}\right]_0^1 =$$

$$-\frac{1}{3} + \frac{a}{3} + \frac{b}{2} = -\frac{1}{3} + \frac{1-2b}{3} + \frac{b}{2} = \frac{-2b}{3} + \frac{b}{2} = 0 \iff$$

$$\frac{-4b+3b}{6} = 0 \iff b = 0 \implies a = 1 - 2b = 1$$

(iii)

$$P(|X| \geq \tfrac{1}{2} \mid X \leq 0) = P(|X| \geq 1/2,\ X \leq 0)$$

$$P(|X| \geq 1/2, X \leq 0) = \frac{P(-1 \leq X \leq -1/2)}{P(-1 \leq X \leq 0)} = \frac{\int_{-1}^{-1/2} (-x)\,dx}{\int_{-1}^{0} (-x)\,dx} = \frac{-\frac{1}{8}+\frac{1}{2}}{\frac{1}{2}} = \frac{3}{4}$$

Es. 3

H_0) $\mu = 30 \qquad n = 16 \qquad \sigma^2 = 36 \qquad I = (26.91, 33.09)$

H_0 è respinta $\iff |\bar{x} - m_0| > \frac{\sigma}{\sqrt{n}}\, q_{1-\alpha/2}$

$$|\bar{x} - 30| > \frac{3}{2}\, q_{1-\alpha/2} \iff 3.09 \cdot \frac{2}{3} = q_{1-\alpha/2} = 2.06$$

$$\Phi(2.06) = 0.98 \to 1 - \alpha/2 = 0.98 \to \alpha = 0.04$$

(ii)

$$C = \left\{ |\bar{x} - 30| > \frac{6}{5} \cdot 2.06 \right\} = \left\{ |\bar{x} - 30| > 2.472 \right\}$$