Es. 1

$\{A, B, C, D, E\} \quad \Omega = 5^3$

$X = \{AAA, BBB, CCC, DDD, EEE\} \quad \# X = 5$

$P_1 = \frac{\# X}{\# \Omega} = \frac{5}{5^3} = \frac{1}{25}$

$Y = \{\text{tutti diversi}\} \quad \# \, 5 \cdot 4 \cdot 3 = 60$

$P_2 = \frac{\# Y}{\# \Omega} = \frac{60}{5^3} = \frac{\cancel{5} \cdot 12}{5^{\cancel{3}2}} = \frac{12}{25}$

$P_3 = 1 - \frac{13}{25} = \frac{12}{25}$

(ii)

$Z = \{1 \text{ in } A\} \quad W = \{2 \text{ in } A\}$

$$P(W|Z) = \frac{P(W \cap Z)}{P(Z)}$$

$\# Z = 125 - 4^3 = 61$

$\# W = 1 + 4 \cdot 3 = 13$

$P(W) = \frac{\# W}{\# \Omega} = 13/61$

Es. 2

$Y = e^X \qquad f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$

$Y = h(X) \qquad h(x) = e^x \qquad \frac{d}{dy} h^{-1}(y) = \frac{1}{y}$

$h^{-1}(y) = \log(y)$

$$f_y(y) = \begin{cases} f_x(\log(y)) \cdot e^y = \lambda e^{-\lambda \log(y)} \cdot \frac{1}{y} = \lambda e^{-\lambda \log(y)} \cdot \frac{1}{y} = \lambda y^{-\lambda - 1} \\ 0 \end{cases}$$

$$E[Y^n] = \int_0^{+\infty} y^n f(y)\, dy = E[e^{nX}] = \int_0^{+\infty} e^{nx} f(x)\, dx = \int_0^{+\infty} \lambda e^{nx - \lambda x}\, dx =$$

$$= \lambda \int_0^{+\infty} e^{x(n-\lambda)}\, dx < +\infty \iff n - \lambda < 0 \iff n < \lambda$$

$$\lambda \left[\frac{e^{(n-\lambda)x}}{n - \lambda} \right]_0^{+\infty} = \frac{\lambda}{\lambda - n} \iff n < \lambda$$

(ii)

$$\int_0^{+\infty} f(t)\, dt = \int_0^{+\infty} \lambda t^{-\lambda - 1}\, dt = \lambda \int_0^{+\infty} t^{-\lambda - 1}\, dt = \lambda \left[\frac{t^{-\lambda}}{-\lambda} \right]_1^y =$$

$$= \lambda \left[-\frac{y^{-\lambda}}{\lambda} \right]_1^y = -y^{-\lambda} + 1 = 1 - y^{-\lambda}$$

(iii)

$F_y(x_\beta) = \beta$

$$1 - x_\beta^{-\lambda} = \beta \iff x_\beta^{-\lambda} = 1 - \beta \iff x_\beta = \sqrt[-\lambda]{1 - \beta}$$

Es. 3

$m = 100 \quad \bar{x} = 2.2 \quad S^2 = 0.9$

(i)

$1-\alpha = 0.95 \quad \alpha = 0.05 \quad 1-\alpha/2 = 0.975$

$$\bar{x} \pm \frac{S}{\sqrt{m}} \cdot \tau_{(0.975, 99)} \iff 2.2 \pm \frac{0.9846}{10} \cdot 1.98535 = 2.2 \pm 0.1954$$

$I = (2.0046, 2.3954) \quad 24 \notin I$

$$P = \frac{S}{\sqrt{m}} \tau_{(1-\alpha/2, m-1)} = 0.1954$$

(ii) $m \leq 0.02 \qquad \sigma = \frac{0.9}{100} = 0.09 \qquad \bar{x} = \frac{2.2}{100} = 0.022$

$$\bar{\alpha} = 1 - \Phi\left(\frac{\sqrt{m}}{\sigma} (\bar{x} - m_0) \right) = 1 - \Phi\left(\frac{10}{0.09} (0.002) \right) = 1 - \Phi(2.22) \sim 0.015$$