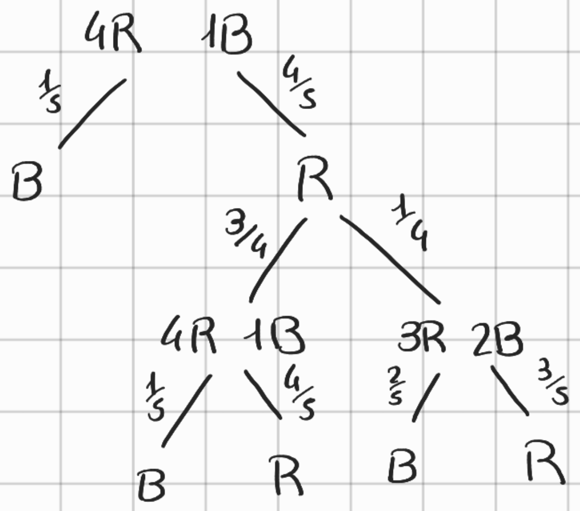
Es. 1



(i)

$P\{\text{estrarre } B\} = P\{\text{prima estratta } B\} + P\{\text{seconda estratta } B\}$

$P\{\text{prima estratta } B\} = \frac{1}{5}$

$P\{\text{seconda estratta } B\} = \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{4}{5} + \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{4}{5} = \frac{1}{5}$

$P\{\text{estrarre } B\} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

$X \sim B(10, \frac{1}{5})$

$P\{X = 3\} = p(3) = \binom{10}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^7 = \frac{10 \cdot 9 \cdot 8}{6} \cdot \frac{1}{125} \cdot \frac{16384}{78125} = \frac{11796480}{58593750}$

~ 0.20132

(ii)

$A = \{\text{seconda estratta } B\}$ $\quad B_1 = \{\text{inserisco } B\}$ $\quad P(B_1) = \frac{1}{4}$

$B_2 = \{\text{inserisco } R\}$ $\quad P(B_2) = 3/4$

$$P(B_1 \mid A) = \frac{P(A \mid B_1)P(B_1)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2)} = \frac{\frac{2}{5} \cdot \frac{1}{4}}{\frac{2}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{3}{4}} =$$

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{3}{20}} = \frac{2}{5}$$

Es. 2

(i)

$X \sim N(1, a)$ Gaussiane $\qquad X = aZ + 1 \qquad Z \sim N(0,1)$

$Var(X) = 4 \iff a^2 = 4 \iff a = 2$

(ii)

$Z = (X - Y)^2 \qquad Y \sim N(1,5) \quad X \sim N(1,4)$

$-Y \sim N(-1,5) \longrightarrow Z \sim (0,9)$

$Z = h(X - Y)$ con $h(t) = t^2$

$$F_Z(z) = P(Z \leq z) = P((X-Y)^2 \leq z) \begin{cases} 0 \\ P(-\sqrt{z} \leq X - Y \leq \sqrt{z}) \end{cases}$$

$V \sim N(0,1) \qquad X - Y = 3V$

$$P(-\sqrt{z} \leq X - Y \leq \sqrt{z}) = P\left(-\frac{\sqrt{z}}{3} \leq V \leq \frac{\sqrt{z}}{3}\right) = \Phi\left(\frac{\sqrt{z}}{3}\right) - \Phi\left(-\frac{\sqrt{z}}{3}\right) =$$

$$= 2\Phi\left(\frac{\sqrt{z}}{3}\right) - 1$$

$$F_Z(z) = \begin{cases} 0 \\ 2\Phi\left(\frac{\sqrt{z}}{3}\right) - 1 \end{cases}$$

$F_Z(z)$ ha densità $f_Z(z) = F_Z'(z)$

$$f_z(z) = \begin{cases} 0 \\ 2\,\varphi\left(\frac{\sqrt{z}}{3}\right) \cdot \frac{1}{6\sqrt{z}} = \ ? \end{cases}$$

(iii)

$Y_m = X_1 + \ldots + X_{100}$

$$P(X_1 + \ldots + X_{100} > 90) = P(Y_m > 90) = P\left(\frac{X_1 + \ldots + X_{100} - m\mu}{\sigma\sqrt{m}} > \frac{90 - 100\mu}{\sigma\sqrt{m}}\right)$$

$$= P\left(\frac{X_1 + \ldots + X_{100} - 100}{20} > \frac{90 - 100}{20}\right) = P\left(Y_1 > -\frac{1}{2}\right) = 1 - \Phi\left(-\frac{1}{2}\right) =$$

$$= 1 - \left(1 - \Phi\left(\frac{1}{2}\right)\right) = \Phi\left(\frac{1}{2}\right) \sim 0.69146$$

Es. 3

$H_0)\ \sigma^2 \leq 0.2 \qquad m = 16 \qquad \bar{X} = 4.935 \qquad S^2 = 0.06$

$S = 0.244948$

$\alpha = 0.05 \qquad 1 - \alpha = 0.95$

$$C = \left\{ \sum_i \frac{(x_i - \bar{x})^2}{\sigma_0^2} > \chi_{(1-\alpha,\, m-1)} \right\}$$

H_0 è accettata $\iff \sum_i \frac{(x_i - \bar{x})^2}{\sigma_0^2} \leq \chi_{(1-\alpha,\, m-1)} \iff \frac{(m-1)\, S^2}{\sigma_0^2} \leq 24.9958$

$\iff \frac{15 \cdot 0.06}{0.04} \leq 24.9958 \iff 22.5 \leq 24.9958$ ✓

H_0 accettate

(ii) $\quad H_0)\ m = 5$

$$C = \left\{ |\bar{x} - m_0| > \frac{S}{\sqrt{m}}\, \tau_{(1-\alpha/2,\, m-1)} \right\}$$

$$\bar{\alpha} = 2\left[1 - F_{n-1}\left(\frac{\sqrt{n}}{S}\,|\bar{x} - m_0|\right)\right] = 2\left[1 - F_{15}\left(\frac{\sqrt{16}}{0.06}\,|4.935 - 5|\right)\right] =$$

$$= 2\left[1 - F_{15}(1.061)\right] = 2 - 2 \cdot 0.825 = 0.35$$

$\bar{\alpha} \sim 0.35 > 0.3 \implies$ Molto plausibile