Es. 1

$A = \{2D \text{ e } 8R\}$ $\quad$ $B = \{5D \text{ e } 5R\}$ $\quad$ X conta i pezzi R

(i)

$X \sim B(3, 4/5)$ $\quad$ $E[X] = \frac{12}{5}$

(ii)

$C_1 = \{\text{Scelgo } A\}$ $\quad$ $C_2 = \{\text{Scelgo } B\}$ $\quad$ $P(C_1) = P(C_2) = 1/2$

$D = \{\text{Prelevo } 3R\}$

$$P(C_1 | D) = \frac{P(D|C_1)\,P(C_1)}{P(D|C_1)P(C_1) + P(D|C_2)P(C_2)} = \frac{\frac{7}{15}\cdot\frac{1}{2}}{\frac{7}{15}\cdot\frac{1}{2} + \frac{1}{12}\cdot\frac{1}{2}} = \frac{28}{33}$$

$$P(D|C_1) = \frac{4}{5}\cdot\frac{7}{9}\cdot\frac{3}{4} = 7/15$$

$$P(D|C_2) = \frac{1}{2}\cdot\frac{4}{9}\cdot\frac{3}{8} = 1/12$$

Es. 2

(i)

$$\int_{\mathbb{R}} f(x)\,dx = 1 \iff \int_0^1 3x^c\,dx = 1 \iff 3\int_0^1 x^c\,dx = 1 \iff$$

$$\iff 3\left[\frac{x^{c+1}}{c+1}\right]_0^1 = 1 \iff \frac{3}{c+1} = 1 \iff c = 2$$

$$E[X] = \int_0^1 x f(x)\,dx = \int_0^1 x\,3x^2\,dx = 3\int_0^1 x^3 = \frac{3}{4}$$

$$Var(X) = E[X^2] - E[X]^2 = \frac{3}{5} - \frac{9}{16} = \frac{48-45}{80} = \frac{3}{80}$$

$$E[X^2] = \int_0^1 3x^4 \, dx = \frac{3}{5}$$

(ii)

$Z = X_1 + \ldots + X_{60}$ $\qquad Y \sim N(0,1)$

$$P(Z \leq 42) = P\left(\frac{X_1 + \ldots + X_{60} - 45}{\sqrt{60 \cdot 3/80}} \leq \frac{42-45}{\sqrt{60 \cdot 3/80}}\right) = P\left(Y \leq -\frac{3}{1.5}\right) = \Phi(-2)$$

$$= 1 - \Phi(2) \sim 0.02275$$

Es. 3

$X_1, \ldots, X_6 \sim N(\mu_1, \sigma_1^2)$

$Y_1, \ldots, Y_6 \sim N(\mu_2, \sigma_2^2)$

$Z_1, \ldots, Z_6$ con $Z_i = X_i - Y_i$ $\quad i = 1, \ldots, 6$

(i)

$Z_i \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

$Z_i = X_i - Y_i$ quindi $-Y_i \sim N(-\mu_2, \sigma_2^2) \rightarrow Z_i = X_i + (-Y_i)$

$\sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

(ii)

$\alpha = 0.05$ $\qquad H_0)\ \mu_1 - \mu_2 = \mu_z$ $\qquad 1 - \frac{\alpha}{2} = 0.975$ $\qquad n = 6$

H_0 accettata se $|\bar{z} - \mu_z| \leq \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)}$

$$\bar{z} = \frac{1}{6} \sum_i x_i = \frac{1}{6}(5 + 4 + 3 + 8 + 0 - 2) = 3$$

$$S^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2 = \frac{1}{5}\left[4 + 1 + 0 + 25 + 9 + 25 \right] = \frac{64}{5}$$

$$\tau_{(0.975,5)} \sim 2.5706$$

$$|3 - 0| \leq \frac{\frac{64}{5}}{\sqrt{6}} \cdot 2.5706 \iff 3 \leq 13.4328 \quad H_0 \text{ accettata}$$

(iii)

$$\bar{\alpha} = 2\left[1 - F_{n-1}\left(\frac{\sqrt{n}}{S} |\bar{z} - \mu_z|\right)\right] \geq 0.3$$

$$2\left[1 - F_5\left(\frac{\sqrt{6}}{\sqrt{\frac{64}{5}}} |3 - \mu_z|\right)\right] \geq 0.3 \iff 2 - 2F_5\left(\frac{\sqrt{6}}{\sqrt{\frac{64}{5}}} |3 - \mu_z|\right) \geq 0.3$$

$$\iff F_5\left(\frac{\sqrt{6}}{\sqrt{\frac{64}{5}}} |3 - \mu_z|\right) \leq 0.85 \iff \frac{\sqrt{6}}{\sqrt{\frac{64}{5}}} |3 - \mu_z| \leq \tau_{(0.85,5)}$$

$$\iff \frac{\sqrt{6}}{\sqrt{\frac{64}{5}}} |3 - \mu_z| \leq 1.1558 \iff |3 - \mu_z| \leq 1.68815$$

$$\mu_z \in (3 - 1.68815,\ 3 + 1.68815)$$