Es. 1

$X_k \sim B(1, p)$

27 volte "0"

63 volte "1"

(i)

$$E[X^k] = \sum_{i=1}^{90} \frac{x_i^k}{n} = \frac{63}{90} = \frac{7}{10} \qquad \hat{p} = 7/10$$

(ii)

$$P\{X_1 + \dots + X_{90} = k\} = \binom{90}{k}\left(\frac{7}{10}\right)^k\left(\frac{3}{10}\right)^{90-k}$$

(iii)

$$P\{X_1 + \dots + X_{90} \geq 60\} \qquad \mu = E[X] = \frac{7}{10} \qquad n = 90$$

$$\sigma^2 = Var(X) = p(1-p) = \frac{7 \cdot 3}{100} = \frac{21}{100}$$

$$P\left\{\frac{X_1 + \dots + X_{90} - 90 \cdot \frac{7}{10}}{\sqrt{90 \cdot \frac{21}{100}}} \geq \frac{60 - 90 \cdot \frac{7}{10}}{\sqrt{90 \cdot \frac{21}{100}}}\right\} = P\left\{Z \geq \frac{60 - 63}{4.347413}\right\} =$$

$$P\left\{Z \geq -\frac{3}{4.347413}\right\} = P\{Z \geq -0.690\} = 1 - \Phi(-0.690) =$$

$$= 1 - (1 - \Phi(0.690)) = \Phi(0.690) \sim 0.7549$$

Es. 2

$f = x \qquad f' = 1$

$g = \qquad g' = e^{-\frac{x^2}{2}}$

(i)

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \iff \int_0^{+\infty} C x e^{-\frac{x^2}{2}}\,dx = C\int_0^{+\infty} x e^{-\frac{x^2}{2}}\,dx = C\left[-e^{-\frac{x^2}{2}}\right]_0^{+\infty} =$$

$$= C = 1$$

$\beta \in (0, 1)$

$$\int_{-\infty}^{x} f(t)\,dt = \begin{cases} 0 \\ 1-e^{-\frac{x^2}{2}} \end{cases}$$

$$\int_{0}^{x} f(t)\,dt = \int_{0}^{x} t\, e^{-\frac{t^2}{2}}\,dt = \left[-e^{-t^2/2}\right]_0^x = 1-e^{-\frac{x^2}{2}}$$

$$F(x_\beta) = \beta \iff 1-e^{-x_\beta^2/2} = \beta \iff e^{-\frac{x_\beta^2}{2}} = 1-\beta \iff -\frac{x_\beta^2}{2} = \log(1-\beta)$$

$$\iff -x_\beta^2 = 2\log(1-\beta) \iff x_\beta = \sqrt{-2\log(1-\beta)}$$

(ii)

$$G_{X^2}(t) = E\left[e^{tX^2}\right] = \int_0^{+\infty} e^{tx^2}\, x\, e^{-x^2/2}\,dx = \int_0^{+\infty} x\, e^{\frac{x^2(2t-1)}{2}}\,dx =$$

$$= \int_0^{+\infty} x\, e^{\frac{x^2}{2}\cdot\frac{(2t-1)}{2}}\,dx = \begin{cases} \infty \\ \frac{1}{2(\frac{1}{2}-t)} \end{cases}$$

Es. 3

$n = 71 \qquad \bar{X} = 140 \qquad S = 0.4$

(i)

$H_0)\ \sigma^2 \geq 0.2 \qquad \alpha = 0.20$

H_0 accettata se $\frac{(n-1)S^2}{\sigma_0^2} \geq \chi^2_{(\alpha, n-1)}$

$\frac{70 \cdot 0.16}{0.2} \geq \chi^2_{(0.20, 70)} \iff 56 \geq 59.8978 \qquad H_0$ non accettata

(ii)

$\bar{\alpha} = G_{n-1}\left(\frac{(n-1)S^2}{\sigma_0^2}\right) = G_{70}(56) \sim 0.125$

$$G_{n-1}\left(\frac{(n-1)\,S^2}{\sigma_0^{\,2}}\right) = 0.25 = G_{70}\left(\frac{70\cdot 0.16}{\sigma_0^{\,2}}\right) = 0.25 \iff \frac{70\cdot 0.16}{\sigma_0^{\,2}} = \chi^2_{(0.25,\,70)}$$

$$\iff \sigma_0^{\,2} \sim 0.1815$$