Es. 1

(i)

$P\{X=1\} = p \rightarrow$ motore guasto $\quad P\{X=0\} = 1-p \rightarrow$ motore integro

Motore bimotore cade $\iff \{X_1 = 1, X_2 = 1\}$

$$P\{X_1=1, X_2=1\} = P\{X_1=1\} P\{X_2=1\} = p^2$$

Motore quadrimotore cade $\iff \{0111, 1011, 1101, 1110, 1111\}$

$$P\{0111, 1011, 1101, 1110, 1111\} = P\{0111\}P\{1011\}P\{1101\}P\{1110\} P\{1111\} = 4p(1-p)p^3 + p^4 = 4p^3 - 4p^4 + p^4 = 4p^3 - 3p^4$$

(ii)

$$p^2 < 4p^3 - 3p^4 \iff p^2 + 3p^4 - 4p^3 < 0 \iff p^2(1 + 3p^2 - 4p) < 0$$

$$\iff 3p^2 - 4p + 1 < 0 \iff p \in (\tfrac{1}{3}, 1) \quad ?$$

Es. 2

(i)

$$\int_{-\infty}^{+\infty} f(x)\, dx = \int_{-1}^{1} cx^2\, dx = c\left[\frac{x^3}{3}\right]_{-1}^{1} = c\left[\frac{2}{3}\right] = \frac{2}{3}c = 1 \iff$$

$$\iff c = \frac{3}{2}$$

$$P\{X \leq x\} = \int_{-\infty}^{x} f(t)\, dt = \int_{-1}^{x} \frac{3}{2}t^2\, dt = \frac{x^3}{2} + \frac{1}{2}$$

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x^3}{2} + \frac{1}{2} & -1 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

(ii)

$\beta \in (0,1)$

$$F(x_\beta) = \beta \iff \frac{x_\beta^3}{2} + \frac{1}{2} = \beta \iff x_\beta^3 = 2\beta - 1 \iff x_\beta = \sqrt[3]{2\beta - 1}$$

$$E[X^n] = \int_{-\infty}^{+\infty} x^n f(x)\,dx = \int_{-1}^{1} x^n \frac{3}{2} x^2\,dx = \frac{3}{2}\int_{-1}^{1} x^{n+2}\,dx = \frac{3}{2}\left[\frac{x^{n+3}}{n+3}\right]_{-1}^{1} =$$

$$= \frac{3}{2}\left[\frac{1}{n+3} - \frac{(-1)^{n+3}}{n+3}\right] = \frac{3(1-(-1)^{n+3})}{2(n+3)} = \begin{cases} 0 & n \text{ dispari} \\ \frac{3}{n+3} & n \text{ pari} \end{cases}$$

(iii)

$Y_n = 2 + X^n$

$$E[Y_n] = 2 + E[X^n] = \begin{cases} 2 \\ 2 + \frac{3}{n+3} \end{cases}$$

$$Var(Y_n) = Var(2 + X^n) = Var(X^n) = E[X^{2n}] - E[X^n]^2 =$$

$$\begin{cases} \frac{3}{2n+3} - 0 = \frac{3}{2n+3} \\ \frac{3}{2n+3} - \left(\frac{3}{n+3}\right)^2 \end{cases}$$

$$E[X^{2n}] = \int_{-1}^{1} x^{2n} \frac{3}{2} x^2\,dx = \frac{3}{2}\int_{-1}^{1} x^{2n+2}\,dx = \frac{3}{2}\left[\frac{x^{2n+3}}{2n+3}\right]_{-1}^{1} =$$

$$= \frac{3}{\cancel{2}}\left[\frac{\cancel{2}}{2n+3}\right] = \frac{3}{2n+3}$$

$$\left.\begin{aligned} &E[Y_n] = 2 \quad n \to +\infty \\ &\lim_{n\to+\infty} Var(Y_n) = 0 \end{aligned}\right\} (Y_n)_{n\geq 1} \text{ converge a } 2$$

Es. 3

$n = 64 \qquad \bar{x} = 47.1 \qquad \sigma = 2.4$

(i)

$H_0)\ m \geq 48$

$\bar{\alpha} = \Phi\left(\frac{\sqrt{n}}{\sigma}(\bar{x} - m_0)\right) = \Phi\left(\frac{8}{2.4}(47.1 - 48)\right) = 1 - \Phi(3) = 1 - 0.99865$

$= 0.00135 < 0.3 \Rightarrow$ poco plausibile

(ii)

$\bar{\alpha} = \Phi\left(\frac{\sqrt{n}}{\sigma}(\bar{x} - m_0)\right) = 0.3 \iff \Phi\left(\frac{8}{2.4}(\bar{x} - 48)\right) = 0.3 \iff$

$\iff \frac{8}{24}(\bar{x} - 48) = 0.3 \iff \frac{8}{2.4}(\bar{x} - 48) = \tau_{(0.3,63)} \sim q_{0.3} = -q_{1-0.3}$

$= -q_{0.7} \sim -0.53 \iff \bar{x} = -0.53 \cdot \frac{24}{8} + 48 = 47.841$