Es. 1

$P\{\text{Non presentarsi}\} = 1/3$

$E_1 = 3$ posti $\quad E_2 = 5$ posti

$\downarrow \qquad \downarrow$

4 prenotazioni $\quad$ 7 prenotazioni

$X_1 \sim B(4, \frac{2}{3})$ $\quad$ X_1 e X_2 contano il numero di prenotazioni

$X_2 \sim B(7, \frac{2}{3})$ $\quad$ $P\{X_1 \geq 4\}$ e $P\{X_2 \geq 6\}$

$$P\{X_1 \geq 4\} = P\{X_1 = 4\} = p(4) = \binom{4}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 = \left(\frac{2}{3}\right)^4$$

$$P\{X_2 \geq 6\} = P\{X_2 = 6\} + P\{X_2 = 7\} = p(6) + p(7) = \binom{7}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right) + \binom{7}{7}\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^0 =$$

$$= 7 \cdot \left(\frac{2}{3}\right)^6 \cdot \left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^6\left(\frac{7}{3} + \frac{2}{3}\right) = \frac{2^6}{3^5}$$

$$P\{X_2 \geq 6\} > P\{X_1 \geq 4\}$$

Es. 2

(i)

$$f(x) = F'(x) = (\theta+1)\, x^\theta$$

$$f(x) = \begin{cases} 0 & x \leq 0 \\ (\theta+1)x^\theta & 0 \leq x \leq 1 \end{cases}$$

X con densità ha $E[X^n] < +\infty$

$$E[X^n] = \int_0^1 x^n f(x)\,dx = \int_0^1 x^n(\theta+1)\,x^\theta\,dx = (\theta+1)\int_0^1 x^{n+\theta}\,dx = (\theta+1)\left[\frac{x^{n+\theta+1}}{n+\theta+1}\right]_0^1$$

$$= (\theta+1)\left[\frac{1}{n+\theta+1}\right] = \frac{\theta+1}{n+\theta+1} < +\infty \quad \forall n$$

(ii)

$$E[X^2] = \frac{1}{2}$$

$$\theta = 1 \longrightarrow E[X] = \frac{2}{3} \qquad Var(X) = E[X^2] - E[X]^2 = \frac{1}{2} - \frac{4}{9} = \frac{9-8}{18} = \frac{1}{18}$$

$$P\{X_1 + \dots + X_{90} \geq 55\} = P\left\{ \frac{X_1 + \dots + X_{90} - 90 \cdot 2/3}{\sqrt{90 \cdot \frac{1}{18}}} \geq \frac{55 - 90 \cdot 2/3}{\sqrt{90 \cdot 1/18}} \right\}$$

Presa $Z = \dfrac{X_1 + \dots + X_{90} - 90 \cdot 2/3}{\sqrt{90 \cdot 1/18}} \sim N(0,1)$

$$P\left\{ Z \geq \frac{55-60}{2.23606} \right\} = 1 - \Phi(-2.23607) = 1 - (1 - \Phi(2.23607)) = \Phi(2.23607)$$

~ 0.987

(iii)

$\bar{X} = \frac{3}{4}$

$$E[X] = 3/4 \iff \int_0^1 x f(x)\, dx = (\theta+1)\int_0^1 x^{\theta+1}\, dx = \theta+1 \left[\frac{x^{\theta+2}}{\theta+2} \right]_0^1 = \frac{\theta+1}{\theta+2}$$

$$\frac{\theta+1}{\theta+2} = 3/4 \iff \theta+1 = \frac{3}{4}(\theta+2) \iff \theta = \frac{3}{4}\theta + \frac{3}{2} - 1 \iff \frac{\theta}{4} = \frac{1}{2}$$

$$\iff \tilde{\theta} = 2$$

Es. 3

$H_0)\ \sigma^2 \leq 4 = \sigma_0^2 \qquad n = 25 \qquad S = 2.3 \qquad \alpha = 0.1 \qquad 1-\alpha = 0.9$

(i)

$C = \left\{ (n-1)\frac{S^2}{\sigma_0^2} > \chi^2_{(1-\alpha, n-1)} \right\}$ H_0 accettata se $(n-1)\frac{S^2}{\sigma_0^2} \leq \chi^2_{(1-\alpha, n-1)}$

$\dfrac{24 \cdot 5.29}{4} \leq \chi^2_{(0.9, 24)} \iff 31.74 \leq 33.1962 \rightarrow H_0$ accettata

(ii)

$$\bar{\alpha} = 1 - G_{m-1}\left((m-1)\frac{S^2}{\sigma_0^{\,2}}\right) = 0.05 \iff G_{24}\left(24 \cdot \frac{S^2}{4}\right) = 0.95 \iff$$

$$\iff S^2 = \chi^2_{(0.95,\,24)} \cdot \frac{1}{6} \iff S = \sqrt{36.4150 \cdot 0.167} \iff S \sim 2.46$$