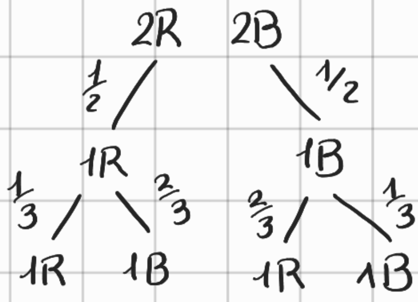
Es. 1

(ii)

2R 2B

X conta il numero di R $\quad K = 0, 1, 2$

$P\{X = K\}$



$$P\{X=0\} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$P\{X=1\} = \left(\frac{1}{\cancel{2}} \cdot \frac{2}{3}\right) \cancel{2} = \frac{2}{3}$$

$$P\{X=2\} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

(ii)

$A = \{$ Estraggo 1R $\}$

$$P\{X=2 \mid A\} = \frac{P\{X=2 \cap A\}}{P(A)} = \frac{P(A \mid X=2)\, P(X=2)}{P(A)} =$$

$$= \frac{P(A \mid X=2)\, P(X=2)}{\sum_{K=0}^{2} P(A \mid A=K)\, P(X=K)} = \frac{1 \cdot \frac{1}{6}}{0 \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{2}{3} + 1 \cdot \frac{1}{6}} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(A \mid A=K) = \begin{cases} 0 \\ \frac{1}{2} \\ 1 \end{cases}$$

Es. 2

F(x) :

1) ✗

2)

3)

G(x) :

1) ✓

2) ✓

3) ✓

$$g'(x) = \frac{4}{x^5} \geq 0 \iff x^{-5} \geq 0 \iff x \geq 0$$

$$\lim_{x \to 1^+} G(x) = G(1) \longrightarrow 1 - 1 = 0 \quad \checkmark$$

$$E[X^k] = \int_{-\infty}^{+\infty} x^k g(x)\, dx = \int_1^{+\infty} x^k \cdot \frac{4}{x^5}\, dx = 4\int_1^{+\infty} x^{k-5}\, dx < +\infty \iff k < 4$$

$$E[X] = \int_1^{+\infty} x\, g(x)\, dx = \int_1^{+\infty} x \cdot \frac{4}{x^5}\, dx = 4\int_1^{+\infty} x^{-4}\, dx = 4\left[-\frac{1}{3x^3}\right]_1^{+\infty} =$$

$$= 4\left(\frac{1}{3}\right) = \frac{4}{3}$$

$$Var(X) = E[X^2] - E[X]^2 = 2 - \frac{16}{9} = \underline{\frac{2}{9}}$$

$$E[X^2] = \int_1^{+\infty} x^2 \cdot \frac{4}{x^5}\, dx = 4\int_1^{+\infty} x^{-3}\, dx = 4\left[-\frac{1}{2x^2}\right]_1^{+\infty} = 4\left(\frac{1}{2}\right) = 2$$

Es.3

$n = 10.000$ 4900 votanti ✓

(i)

$\hat{p} = \frac{4900}{10000} = 0.49$ $X = 0 \to NO$ $X = 1 \longrightarrow SI$

$X_1, \dots, X_{10000}$ $X_i \sim B(1, p)$ $X \sim B(4900, p)$

$$\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \cdot q_{1-\frac{\alpha}{2}} = 10^{-3} \iff q_{1-\frac{\alpha}{2}} = 10^{-3} \frac{\sqrt{10.000}}{\sqrt{(0.49)(1-0.49)}} \iff$$

$$1 - \frac{\alpha}{2} = \Phi\left(\frac{0.1}{0.4999}\right) \iff \alpha = 2 - 2\Phi(0.2) = 2 - 2 \cdot 0.57926 = 0.84148$$

$$1 - \alpha = 1 - 0.84148 \sim 0.15852$$

(ii)

$\mathcal{H}_0)\ p \geq 0.5$

$$\bar{\alpha} = \Phi\left(\frac{\sqrt{n}(\hat{p}-p_0)}{\sqrt{p_0(1-p_0)}}\right) = \Phi\left(\frac{100\,(-0.01)}{\sqrt{\frac{1}{4}}}\right) = 1-\Phi(2) = 1-0.97725 = 0.02275$$

$\bar{\alpha} < 0.3 \longrightarrow$ poco plausibile