1) $\Omega = \{RRR, RRB, RBR, BRR, RBB, BRB, BBR, BBB\}$
$\#\Omega = 8$

$A = \{\text{Almeno 1 rossa}\}$ $A^c = \{BBB\}$ $\#A = \#\Omega - \#A^c = 8 - 1 = 7$

$B = \{\text{Almeno 1 blu}\}$ $B^c = \{RRR\}$ $\#B = \#\Omega - \#B^c = 8 - 1 = 7$

$P(A) = \frac{7}{8}$ $P(A \cap B) = \{\text{Almeno 1R e 1B}\}$ $(A \cap B)^c = \{RRR, BBB\}$
$\#(A \cap B) = \#\Omega - \#(A \cap B)^c = 8 - 2 = 6$

(i)
$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{6}{8} \cdot \frac{8}{7} = \frac{6}{7}$

(ii) $R \to \frac{2}{3}$ $B \to \frac{1}{3}$
$P(A) = 1 - P(A^c) = 1 - P(\{BBB\}) = 1 - \left(\frac{1}{3}\right)^3 = 26/27$
$P(B) = 1 - P(B^c) = 1 - P(\{RRR\}) = 1 - \left(\frac{2}{3}\right)^3 = 19/27$
$P(A \cap B) = 1 - (P(A^c) + P(B^c)) = 1 - \left(\frac{1}{27} + \frac{8}{27}\right) = 1 - \frac{9}{27} = 2/3$
$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^{1}\cancel{2}/\cancel{3}}{} \cdot \frac{\cancel{27}^{9}}{\cancel{26}_{13}} = \frac{9}{13}$

2) $\Omega = \{(1,1), \ldots, (6,6)\}$ $\#\Omega = 6^2 = 36$

$A = \{\text{numero dispari}\}$ $B = \{\text{numero pari}\}$
$\#A = 3$ $\#B = 3$

$C = \{\text{Somma pari}\} = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,3),(1,5),$
$(2,4),(2,6),(3,1),(3,5),(4,2),(4,6),(5,1),(5,3),$
$(6,2),(6,4)\}$ $\#C = 18$

$P(A) = P(B) = \frac{1}{2}$
$P(C) = \frac{\#C}{\#\Omega} = \frac{{}^{1}\cancel{18}}{\cancel{36}_{2}} = \frac{1}{2}$

$P(A \cap B) = \{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}$ $\#(A \cap B) = 9$

$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ o $\frac{9}{36_4}^{1}$

$P(A \cap C) = \{(1,1),(3,3),(5,5),(1,3),(1,5),(3,1),(3,5),(5,1),(5,3)\}$ $\#A \cap C = 9$

$P(A \cap C) = P(A) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ o $\frac{9}{36_4}^{1}$

$P(B \cap C) = \{(2,2),(4,4),(6,6),(2,4),(2,6),(4,2),(4,6),(6,2),(6,4)\}$ $\#(B \cap C) = 9$

$P(B \cap C) = P(B) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ o $\frac{9}{36_4}^{1}$

$P(A \cap B \cap C) = 0 \neq P(A) \cdot P(B) \cdot P(C)$

A, B, C dipendenti

A e B
B e C
A e C
} Indipendenti

3) | 5R 7B | S1 | | 8R 3B | S2 |

$B1$ = Scelgo S1 $\quad P(B1) = P(B2) = \frac{1}{2}$
$B2$ = Scelgo S2

$A = \{$ Estraggo 2R $\}$

$P(A \mid B1) = \frac{P(A \cap B1)}{P(B1)} = \frac{\frac{1}{2}\left(\frac{5}{12} \cdot \frac{4}{11}\right)}{\frac{1}{2}} = \frac{\cancel{20}^{10}}{\cancel{132}_{66}^{5}_{33}} = \frac{5}{33}$

$$P(A|B_2) = \frac{P(A \cap B_2)}{P(B_2)} = \frac{\frac{1}{2}\left(\frac{8}{11}\cdot\frac{7}{10}\right)}{\frac{1}{2}} = \frac{\cancel{56}^{28}}{\cancel{110}_{55}} = \frac{28}{55}$$

$$P(B_1|A) = \frac{P(A|B_1)\cdot P(B_1)}{P(A|B_1)\cdot P(B_1) + P(A|B_2)\cdot P(B_2)} = \frac{5/66}{5/66 + \frac{28}{110}} = \frac{25}{109}$$

④ 2R 3B (S1) 3R 4B (S2)

$A = \{\text{sposto } R\}$ $P(A) = \frac{2}{5}$ $P(A^c) = \frac{3}{5}$

$B = \{\text{estraggo 2R da S2}\}$

$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$

$$P(B|A) = \frac{\binom{4}{2}}{\binom{8}{2}} = \frac{6}{28} = \frac{3}{14}$$

$$P(B|A^c) = \frac{\binom{3}{2}}{\binom{8}{2}} = \frac{3}{28} = \frac{3}{28}$$

$$P(B) = \frac{3}{14}\cdot\frac{2}{5} + \frac{3}{28}\cdot\frac{3}{5} = \frac{3}{20}$$

⑤ $\Omega = \{VVVV, VVVF, \ldots, FFFF\}$ $\#\Omega = 2^4 = 16$

$P(\{V\}) = 2/3$

$P(\{F\}) = 1/3$

(i)

$$P(\Omega - \{FFFF\}) = 1 - P(\{FFFF\}) = 1 - \left(\frac{1}{3}\right)^4 = 1 - \frac{1}{81} = 80/81$$

(ii)

$$P(\{VFFF, FVFF, FFVF, FFFV\}) = 4\cdot P(\{VFFF\}) = 4\cdot\frac{2}{3}\cdot\left(\frac{1}{3}\right)^3 = \frac{8}{81}$$

(iii)

$$P(\{VVFF, VFVF, VFFV, FVFV, FFVV, FVVF\}) = 6\cdot P(\{VVFF\}) = 6\cdot\left(\frac{2}{3}\right)^2\cdot\left(\frac{1}{3}\right)^2 = \frac{24}{81}$$

6) $A = \{\text{Test positivo}\}$ $B_1 = \{\text{Prodotto integro}\}$ $B_2 = \{\text{Prodotto difettoso}\}$

$$P(B_1) = \frac{99}{100} \qquad P(B_2) = \frac{1}{100}$$

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) = \frac{1}{10}\cdot\frac{99}{100} + \frac{95}{100}\cdot\frac{1}{100} = \frac{1085}{10000}$$

(i)

$$P(B_1|A) = \frac{P(A|B_1)\,P(B_1)}{P(A)} = \frac{1}{10}\cdot\frac{99}{100}\cdot\frac{10000}{1085} = \frac{990}{1085}$$

(ii)

$$P(B_1|\bar{A}) = \frac{P(\bar{A}|B_1)\,P(B_1)}{P(\bar{A})} = \frac{90}{100}\cdot\frac{99}{100}\cdot\frac{10000}{8915} = \frac{8910}{8915} = \frac{1782}{1783}$$

$$P(A^c) = P(A^c|B_1)\,P(B_1) + P(A^c|B_2)\,P(B_2) = \frac{90}{100}\cdot\frac{99}{100} + \frac{5}{100}\cdot\frac{1}{100} = \frac{8915}{10000}$$