Es. 1

(i)

$P\{1 \text{ guasto}\} = 0.3$ 2 su 5 per funzionare $n = 5$

$X \sim B(5, 7/10)$ $P\{X \geq 2\} = p(2) + p(3) + p(4) + p(5)$

$$P\{X \geq 2\} = \binom{5}{2}\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3 + \binom{5}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2 + \binom{5}{4}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right) + \binom{5}{5}\left(\frac{7}{10}\right)\left(\frac{3}{10}\right)^0 =$$

$$= 10 \cdot \left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3 + 10 \cdot \left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2 + 5\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right) + \frac{7}{10} = 0.96922$$

(ii)

$A = \{\text{Funziona}\}$ $P(A) = 0.96922$

$X_1 = 0$ se la prima è guasta

$X_1 = 1$ " " funzionante

$$P\{Y = 0 \mid A\} = \frac{P\{X_1 = 0 \cap A\}}{P(A)} = \frac{P(A \mid X_1 = 0)}{P(A)} \cdot P(X_1 = 0) =$$

$$= \frac{P(X_2 + X_3 + X_4 + X_5 \in \{2,3,4\})}{P(A)} \cdot P(X_1 = 0)$$

$$Z = X_2 + X_3 + X_4 + X_5 \sim B(4, p)$$

$$P(Z \geq 2) = p(2) + p(3) + p(4) + p(5) = \frac{9163}{10\,000} = 0.9163$$

$$P(X_1 = 0 \mid A) = \frac{9163}{\cancel{10\,000}} \cdot \frac{3}{10} \cdot \frac{\cancel{10\,000}}{96922} \sim 0.283$$

Es. 2

(i)

- $\lim_{x \to -\infty} F(x) = 0$ e $\lim_{x \to +\infty} F(x) = 1$ ✓
- $F'(x) \geqslant 0 \iff a + 2xb \geqslant 0$ ✓
- $\lim_{x \to 1^+} F(x) = F(1) \iff a + b = 1 \longrightarrow b = 1 - a$

(ii)

$$E[X] = \frac{1}{2} \iff \int_0^1 x f(x)\, dx = \int_0^1 ax + 2x^2 b \, dx = \frac{a}{2} + 2b \left[\frac{x^3}{3} \right]_0^1 =$$

$$= \frac{a}{2} + \frac{2b}{3} = \frac{1}{2} \iff \frac{a}{2} + \frac{2-2a}{3} = \frac{1}{2} \iff -\frac{a}{6} + \frac{2}{3} = \frac{1}{2}$$

$$\iff a = 6\left(\frac{2}{3} - \frac{1}{2}\right) \iff a = 1 \qquad b = 0$$

$$E[X] = -1 \iff a = 6\left(\frac{2}{3} + 1\right) \iff a = 10 \qquad b = -9 \qquad \text{NO}$$

(iii)

$$Y = \log(X) \qquad h^{-1}(y) = e^y \qquad \frac{d h^{-1}(y)}{dy} = e^y$$

$$f_Y(y) = \begin{cases} f_X(h^{-1}(y)) \cdot \left| \dfrac{d h^{-1}(y)}{dy} \right| = (a + 2b e^y)\, e^y \\ 0 \end{cases}$$

Es. 3

$n = 81 \qquad \bar{x} = 235.44 \qquad \sigma^2 = 3.6^2 = 12.96 \qquad d = 0.82$

$$d = \frac{\sigma}{\sqrt{n}} \, q_{1-\frac{\alpha}{2}}$$

$$\frac{\sigma}{\sqrt{n}} q_{1-\alpha/2} = 0.82 \iff q_{1-\alpha/2} = 0.82 \cdot \frac{\sqrt{81}}{3.6} \iff 1-\frac{\alpha}{2} = \Phi(2.05)$$

$$\iff 2(1-\Phi(2.05)) = \alpha \iff \alpha = 2(1-0.97982) \iff \alpha = 0.04$$

$1-\alpha \sim 0.96$

(ii)

$H_0)\ \sigma^2 \leq 100 = \sigma_0^2$

$$\bar{\alpha} = 1 - G_{n-1}\left((n-1)\frac{S^2}{\sigma_0^2}\right) = 1 - G_{n-1}\left(80 \cdot \frac{12.96}{10}\right) = 1 - G_{80}(103.86) \sim 0.04$$