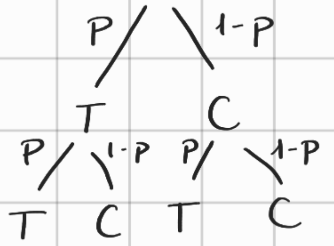
Es.1

(i)

$P(T) = p \qquad P(C) = 1-p$



$P(TT) = p^2 = P(TC) + P(CT)$

$p^2 = p(1-p) + p(1-p) \iff p^2 = p - p^2 + p - p^2 \iff 3p^2 + 2p = 0$

$\iff p(3p+2) = 0 \; \begin{cases} p = 0 \\ 3p+2 = 0 \iff p = 2/3 \end{cases}$

(ii)

$X \sim B(450, 2/3)$ conta il numero di T

$P(X \geq 290) = P\left(Z \geq \frac{290-300}{10}\right) = P(Z \geq -1) = \Phi(1) \sim 0.84134$

Es.2

(i)

- $\lim_{x \to -\infty} F(x) = 0$ e $\lim_{x \to +\infty} F(x) = 1$ ✓
- $F'(x) \geq 0 \iff 2cx \geq 0 \iff c \geq 0$ ✓

$F(0) \geq 0 \iff 0 \geq 0$ ✓ e $F(\sqrt{\theta}) \leq 1 \iff 1 \leq 1$ ✓

- $\lim_{x \to 0^+} F(x) = 0 \iff c \cdot 0^2 = 0$ ✓ e $\lim_{x \to \sqrt{\theta}^-} F(x) = 1 \iff c\theta = 1$

$\iff c = 1/\theta$

$$f(x) = \begin{cases} \frac{2}{\theta}x & 0 \leq x < \sqrt{\theta} \\ 0 & x \geq \sqrt{\theta} \end{cases}$$

$$E[X] = \int_0^{\sqrt{\theta}} x\, f(x)\, dx = \frac{2}{\theta}\int_0^{\sqrt{\theta}} x^2\, dx = \frac{2}{\theta}\left[\frac{x^3}{3}\right]_0^{\sqrt{\theta}} = \frac{2}{\theta}\cdot\frac{\theta^{3/2}}{3} = \frac{2\sqrt{\theta}}{3}$$

$$Var(X) = E[X^2] - E[X]^2 = \frac{1}{2}\theta - \frac{4}{9}\theta = \frac{9-8}{18} = \frac{1}{18}\theta$$

$$E[X^2] = \frac{2}{\theta}\int_0^{\sqrt{\theta}} x^3\, dx = \frac{2}{\theta}\left[\frac{x^4}{4}\right]_0^{\sqrt{\theta}} = \frac{\cancel{2}}{\cancel{\theta}}\cdot\frac{\theta^{\cancel{2}}}{\cancel{4}_2} = \frac{1}{2}\theta$$

(ii)

$$f(x) = \begin{cases} \frac{2}{\theta}x & 0 \le x < \sqrt{\theta} \\ 0 & x \geq \sqrt{\theta} \end{cases}$$

- <u>Max verosimiglianza</u>

$$L(\theta, x_1, \dots, x_n) = \prod_{i=1}^{n} f_\theta(x_i) = \frac{2^n}{\theta^n}\prod_{i=1}^{n} x_i$$

- <u>Momenti</u>

$$E[X] = \bar{X} \iff \frac{2}{3}\sqrt{\theta} = \bar{x} \iff \sqrt{\theta} = \frac{3}{2}\bar{x} \iff \hat{\theta} = \frac{9}{4}\bar{x}^2$$

<u>Es. 3</u>

$$n = 200 \qquad \bar{X} = \frac{8}{200} = \hat{p} = 0.04$$

(i)

$d = 0.02$

$$\frac{\sqrt{0.04(0.96)}}{\sqrt{200}}\, q_{1-\alpha/2} = 0.02 \iff q_{1-\alpha/2} = 0.02 \cdot \frac{\sqrt{200}}{\sqrt{0.0384}} \iff$$

$$\iff q_{1-\alpha/2} = 0.02 \cdot \frac{14.142136}{0.196} = 1.443 \iff 1-\alpha/2 = \Phi(1.443) \iff$$

$$\iff 1 - 0.92507 = \frac{\alpha}{2} \iff \alpha = 0.14986 \longrightarrow 1-\alpha = 0.85$$

$I = \bar{x} \pm d \qquad 1 - \alpha/2 = 0.925$

$$d = \frac{\sqrt{0.04(0.96)}}{\sqrt{200}} \cdot q_{(0.925)} = 0.01386 \cdot 1.44 = 0.02$$

$I = (0.02, 0.06)$

(ii)

$H_0)\ p \leq 0.02 = p_0$

$$\bar{\alpha} = 1 - \Phi\left(\frac{\sqrt{n}\,(\hat{p} - p_0)}{\sqrt{p_0(1-p_0)}}\right) = 1 - \Phi\left(\frac{\sqrt{200}\cdot(0.04 - 0.02)}{\sqrt{0.02 \cdot (0.98)}}\right) = 1 - \Phi\left(\frac{0.283}{0.14}\right) =$$

$= 1 - \Phi(2.02) = 1 - 0.97831 = 0.022 < 0.3$ poco plausibile

(iii)

$$\bar{\alpha} = 1 - \Phi\left(\frac{\sqrt{n}(\hat{p} - p_0)}{\sqrt{p_0(1-p_0)}}\right) \geq 0.3 \iff 0.7 \geq \Phi\left(\frac{\sqrt{200}\,(\hat{p} - 0.02)}{0.14}\right) \iff$$

$$\iff \frac{\sqrt{200}\,(\hat{p} - 0.02)}{0.14} \leq q_{0.7} \iff \hat{p} \leq 0.525 \cdot \frac{0.14}{14.1421} + 0.02 \iff$$

$\iff \hat{p} \leq 0.025$