Es. 1

(i) X ha parametro $p = \frac{9}{10}$ $\quad (1-p) = \frac{1}{10}$

$P\{X=1\} = \frac{9}{10}$ $\quad P\{X=2\} = \frac{9}{10} \cdot \frac{1}{10}$

$p(h) = (1-p)^{h-1} \cdot p = \frac{1}{10^{h-1}} \cdot \frac{9}{10}$

(ii) Y ha parametro $p = \frac{8}{10}$ $\quad (1-p) = \frac{2}{10}$

$P\{Y=1\} = \frac{8}{10}$ $\quad P\{Y=2\} = \frac{2}{10} \cdot \frac{8}{10} = \frac{16}{100} = \frac{4}{25}$

$p(h) = \left(\frac{2}{10}\right)^{h-1} \cdot \frac{8}{10} = \frac{1}{5^{h-1}} \cdot \frac{4}{5}$

(iii) $P\{X+Y=3\} = ?$ $\quad X = \overbrace{3-Y}^{K}$ $\quad 3-Y=k$

$Y = 3-k$

$$\sum_{k=0}^{3} P\{X=K\} P\{Y=3-k\} = \sum_{x=1}^{2} P\{X=k\} P\{Y=3-k\}$$

$$= P\{X=1\} P\{Y=2\} + P\{X=2\} P\{Y=1\} = \frac{9}{10} \cdot \frac{4}{25} + \frac{9}{100} \cdot \frac{8}{10} =$$

$$= \frac{36}{250} + \frac{72}{1000} = \frac{27}{125}$$

Es. 2

$$f(x) = F'(x) = \begin{cases} 0 & x < 1 \\ \frac{3}{x^4} & x \geq 1 \end{cases}$$

$$E[X] = \int_1^{+\infty} \frac{3}{x^3}\, dx = 3\int_1^{+\infty} x^{-3}\, dx = 3\left[-\frac{1}{2x^2}\right]_1^{+\infty} = 3\left[\frac{1}{2}\right] = \frac{3}{2}$$

$$Var(X) = E[X^2] - E[X]^2 = 3 - \frac{9}{4} = \frac{3}{4}$$

$$E[X^2] = \int_1^{+\infty} \frac{3}{x^2}\,dx = 3\int_1^{+\infty} x^{-2}\,dx = 3\left[-\frac{1}{x}\right]_1^{+\infty} = 3$$

$$E[X^3] = \int_1^{+\infty} \frac{3}{x}\,dx = 3\int_1^{+\infty} x^{-1}\,dx = 3\left[1\right]_1^{+\infty} \rightarrow \text{non esiste}$$

(ii)

$$E[(X-Y)^2] = E[X^2 - 2XY + Y^2] = E[X^2] - 2E[XY] + E[Y^2] =$$

$$= 6 - 2(E[X]\cdot E[Y]) = 6 - 2\left(\frac{9}{4}_2\right) = 6 - \frac{9}{2} = \frac{3}{2}$$

(iii)

$n = 80$

$$P\{X_1 + \dots + X_{80} \leq 100\} = P\left\{\frac{X_1 + \dots X_{80} - 120}{\left(\frac{3}{4}\right)^2\sqrt{80}} \leq \frac{100-120}{\left(\frac{3}{4}\right)^2\sqrt{80}}\right\} =$$

$$= P\left\{\frac{X_1 + \dots + X_{80} - 120}{\sqrt{60}} \leq -\frac{20}{\sqrt{60}}\right\} = P\left\{Z \leq -\frac{20}{\sqrt{60}}\right\} = 1 - \Phi\left(\frac{20}{\sqrt{60}}\right) =$$

$$= 1 - \Phi(2.5819) = 1 - 0.99506 = 0.00494$$

Es. 3

(i)

$1-\alpha = 0.90 \rightarrow \alpha = 0.10$ $\quad \hat{p} = \bar{x} = 0.23$ $\quad n = 192$

$1 - \frac{\alpha}{2} = 0.95$ $\quad q_{1-\alpha/2} = q_{0.95} \sim 1.645$

$$s = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}\, q_{1-\alpha/2} = \frac{\sqrt{0.23(0.77)}}{\sqrt{192}} \cdot 1.645 \sim 0.0499$$

(ii)

$H_0) \; p \geq 0.25$ $\quad n = 192$ $\quad \bar{x} = 0.23$ $\quad s^2 =$

$$\alpha = \Phi\left(\frac{\sqrt{n}}{\sqrt{p_0(1-p_0)}}(\hat{p} - p_0)\right) = \Phi\left(\frac{\sqrt{192}}{\sqrt{0.25(0.75)}}(-0.02)\right) = 1 - \Phi(0.64) = 0.26109$$