Es. 1

$X \sim B(10, 1/4)$ $\qquad p(h) = P\{X = h\} = \binom{n}{h} p^h (1-p)^{n-h}$

(i) $P\{X \geq 8\} = ?$

$p(8) = \binom{10}{8}\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 = 45 \cdot \frac{9}{1048576} = \frac{405}{1048576}$

$p(9) = 10\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right) = \frac{30}{1048576}$

$p(10) = \left(\frac{1}{4}\right)^{10} = \frac{1}{1048576}$

$P\{X \geq 8\} = p(8) + p(9) + p(10) = \frac{436}{1048576} \sim 0.0004$

(ii) $E[Y] = ?$ $\qquad X = \#$ risposte corrette $\qquad (10 - X) = \#$ risposte sbagliate

$Y = (+1)X - \frac{1}{4}(10 - X) = X + \frac{X}{4} - \frac{10}{4} = \frac{5X - 10}{4}$

$E[Y] = E\left[\frac{5X-10}{4}\right] = E\left[\frac{5}{4}X - \frac{10}{4}\right] = \frac{5}{4}E[X] - \frac{5}{2} =$

$= \frac{5}{4}\underbrace{\left(10 \cdot \frac{1}{4}\right)}_{E[X] = n \cdot p} - \frac{5}{2} = \frac{\cancel{50}^{\,25}}{\cancel{16}_{\,8}} - \frac{5}{2} = \frac{25 - 20}{8} = 5/8$

Es. 2

(i) a, b ?

1) $\lim\limits_{x \to -\infty} F(x) = 0$ e $\lim\limits_{x \to +\infty} F(x) = 1$ ✓

2) F continua ✓

3) F debolmente crescente ✓

2) $\lim_{x \to 0^+} F(x) = F(0) \to a - 1 = 0 \iff \boxed{a = 1}$

$\lim_{x \to 1^+} F(x) = F(1) \longrightarrow a - e^{-1} = a - e^{-1}$

$\lim_{x \to 2^+} F(x) = F(2) \to 1 = a - e^{-1} + b \iff \boxed{b = e^{-1}}$

$$F'(x) = \begin{cases} 0 \\ e^{-x} \\ e^{-1} \\ 0 \end{cases} \qquad e^{-x} \geq 0 \iff \frac{1}{e^{x}} \geq 0 \quad \forall x$$

(ii)

$\forall \beta \in (0,1) \ \exists! \ x_\beta$ t.c. $F(x_\beta) = \beta$

Se $\beta \in (0, 1 - e^{-1}] \longrightarrow F(x_\beta) = \beta \iff 1 - e^{-x_\beta} = \beta \iff e^{-x_\beta} = 1 - \beta$

$\iff x_\beta = -\log(1 - \beta)$

Se $\beta \in [1 - e^{-1}, 1) \longrightarrow F(x_\beta) = \beta \iff 1 - e^{-1} + e^{-1}(x_\beta - 1) = \beta \iff$

$\iff \frac{x_\beta}{e} = \beta + 2e^{-1} - 1 \iff x_\beta = e\beta + 2 - e \iff$

$\iff x_\beta = e(\beta - 1) + 2$

$$x_\beta = \begin{cases} -\log(1-\beta) & \beta \in (0, 1 - e^{-1}] \\ e(\beta - 1) + 2 & \beta \in [1 - e^{-1}, 1) \end{cases}$$

(iii) $f_Y(y) = ?$ $E[Y] = ?$ $Y = (X+1)^2$

$Y = h \circ g$ $g(x) = x+1$ $h(x) = x^2$

$g^{-1}(x) = x-1$ $h^{-1}(y) = \sqrt{y}$

$$f_{g(x)}(x) = f_x(x-1)$$

$$f_Y(y) = f_{g(x)}(\sqrt{y}) \left| \frac{1}{2\sqrt{y}} \right| = f_x(\sqrt{y}-1) \cdot \left| \frac{1}{2\sqrt{y}} \right| = \frac{e^{-\sqrt{y}+1}}{2\sqrt{y}}$$

$$f_Y(y) = \begin{cases} \frac{e^{-\sqrt{y}+1}}{2\sqrt{y}} \\ \frac{1}{2\sqrt{y}} \\ 0 \end{cases}$$

$$E[Y] = E[(X+1)^2] = E[X^2] + 2E[X] + 1$$

$$E[X] = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_0^1 x e^{-x} dx + \int_1^2 x e^{-1} dx =$$

$$= \left[-(x+1)e^{-x}\right]_0^1 + \left[\frac{x^2}{2} e^{-1}\right]_1^2 = 1 - \frac{1}{2} e^{-1}$$

$$E[X]^2 = \int_{-\infty}^{+\infty} x^2 f(x)\, dx = \int_0^1 x^2 e^{-x} dx + \int_1^2 x^2 e^{-1} dx =$$

$$= \left[-(x^2+2x+2)e^{-x}\right]_0^1 + \left[\frac{x^3}{3} e^{-1}\right]_1^2 = -\frac{8}{3} e^{-1} + 2$$

$$E[Y] = -\frac{8}{3} e^{-1} + 2 + 2\left(1 - \frac{1}{2} e^{-1}\right) + 1 = 5 - \frac{11}{3} e^{-1}$$

Es. 3 $S = \sqrt{S^2}\ \sqrt{0.49} = 0.7$ $1-\alpha = 0.90 \rightarrow \alpha = 0.10 \rightarrow \frac{\alpha}{2} = 0.05$

$n = 10$ $\bar{x} = 4.8$ $1 - \alpha/2 = 0.995$

(i) $= I = \bar{x} + \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2, n-1)}$

$$I_d = \bar{x} + \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)} = 4.8 + \frac{0.7}{\sqrt{10}} \tau_{(0.995,\, 9)} = 5.206$$

$$I_s = \bar{x} - \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)} = 4.8 - \frac{0.7}{\sqrt{10}} \tau_{(0.995,\, 9)} = 4.394$$

$$I = [4.394 , 5.206]$$

$$\frac{\tau_{(1-\alpha/2,\, n-1)} \frac{S}{\sqrt{n}}}{|\bar{x}|} = \frac{\tau_{(1-\alpha/2,\, 9)} \frac{0.7}{\sqrt{10}}}{4.8}$$

$$\frac{\tau_{(1-\alpha/2,\, 9)} \frac{0.7}{\sqrt{10}}}{4.8} \sim 5 \cdot 10^{-2} \iff \tau_{(1-\alpha/2,\, 9)} \sim \frac{4\sqrt{10}}{0.7} \cdot 5 \cdot 10^{-2} \sim 1.084$$

$$\iff 1 - \frac{\alpha}{2} \sim F_9(1.084) \sim 0.84 \iff \alpha \sim 2 \cdot 0.16 \iff$$

$$1 - \alpha \sim 0.68$$

(ii)

$H_0)$ $m \leq 4.3$ $\qquad \bar{\alpha} = 1 - \Phi\left(\frac{\sqrt{n}}{\sigma}(\bar{x} - m_0)\right)$

$$\bar{\alpha} = 1 - \Phi\left(\frac{\sqrt{10}}{0.7}(4.8 - 4.3)\right) = 1 - \Phi\left(\frac{3.16227}{0.7} \cdot 05\right) = 1 - \Phi(2.2587)$$

$\sim 1 - 0.987935 = 0.012065$ $\qquad$ poco plausibile $(\bar{\alpha} < 0.3)$