Es. 1

(i)

$T = \{ \text{Tradizionali} \}$ $NT = \{ \text{Non - T} \}$ $P(T) = 2/3$ $P(NT) = 1/3$

$G = \{ \text{Guarire} \}$ $P\{G|T\} = 0.36$ $P\{G\} = 0.41$

$$P(G) = P(G|T)P(T) + P(G|NT)P(NT)$$

$$P(G|NT) = \frac{P(G) - P(G|T)P(T)}{P(NT)} = \frac{0.41 - (0.36 \cdot 2/3)}{\frac{1}{3}} = 0.51$$

(ii)

$$P(T|G) > P(NT|G)$$

$$P(T|G) = \frac{P(G|T)P(T)}{P(G)} = \frac{0.36 \cdot 2/3}{0.41} = 0.5854$$

$$P(NT|G) = \frac{P(G|NT)P(NT)}{P(G)} = \frac{0.51 \cdot 1/3}{0.41} = 0.4146$$

Es. 2

- $\lim_{x \to -\infty} F(x) = 0$ $\lim_{x \to +\infty} F(x) = 1$
- $F'(x) \geq 0 \iff 2x \geq 0 \iff x \geq 0 \to a \geq 0$
- $\lim_{x \to a^+} F(x) = 0 \iff a^2 - 1 = 0$, $a = 1$ $\lim_{x \to b^-} F(x) = 1 \iff b^2 - 1 = 1$, $b = \sqrt{2}$

(ii) $Var(X) = E[X^2] - E[X]^2 = \frac{3}{2} - \left(\frac{2^{5/2} - 2}{3}\right)^2 = 0.01416$

$f(x) = \begin{cases} 2x \\ 0 \end{cases}$

$E[X] = \int_1^{\sqrt{2}} x f(x) = 2\int_1^{\sqrt{2}} x^2 = 2\left[\frac{x^3}{3}\right]_1^{\sqrt{2}} =$

$= 2\left[\frac{2^{3/2}}{3} - \frac{1}{3}\right] = \frac{2^{5/2}}{3} - \frac{2}{3} = \frac{2^{5/2} - 2}{3}$

$E[X^2] = \int_1^{\sqrt{2}} 2x^3 = 2\left[\frac{x^4}{4}\right]_1^{\sqrt{2}} = 2\left[2 - \frac{1}{4}\right] = 2 - \frac{1}{2} = 3/2$

$P\{X_1 + \ldots + X_{90} \geq 108\} = P\left\{\frac{X_1 + \ldots + X_{90} - 90 \cdot \left(\frac{2^{5/2} - 2}{3}\right)}{\sqrt{90 \cdot 0.01416}} \geq \frac{108 - 90\left(\frac{2^{5/2} - 2}{3}\right)}{\sqrt{90 \cdot 0.01416}}\right\}$

$P\left\{Z \geq -\frac{1.7057}{1.1289}\right\} = \Phi(1.51) \sim 0.9346$

Es. 3

$H_0)\ \sigma^2 \leq 0.1 = \sigma_0^2 \qquad n = 41 \qquad S^2 = 0.145 \qquad \alpha = 0.05$

$1 - \alpha = 0.95$

(i)

H_0 accettata se $(n-1)\frac{S^2}{\sigma_0^2} \leq \chi^2_{(1-\alpha, n-1)}$

$\frac{40 \cdot 0.145}{0.1} \leq \chi^2_{(0.95, 40)} \iff 58 \leq 55.7585$ H_0 non accettata

$\bar{\alpha} < \alpha = 0.05$

(ii)

$\frac{40 \cdot S^2}{0.1} \leq \chi^2_{(0.9, 40)} \iff S^2 \leq \frac{51.8050}{400} \iff S^2 \leq 0.13$