

Dimostrazione della legge tre fattoriali

Caso base

$$\binom{n}{0} = \frac{n!}{0!n!} = 1$$

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Passo induttivo

$$\begin{aligned}\binom{n}{k} &= \binom{n-1}{k} + \binom{n-1}{k-1} \\&= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\&= \frac{(n-1)!}{(k-1)!(n-1-k)!} \cdot \left(\frac{1}{k} + \frac{1}{n-k} \right) \\&= \frac{(n-1)!}{(k-1)!(n-1-k)!} \cdot \frac{n}{k(n-k)} \\&= \frac{n!}{k!(n-k)!}\end{aligned}$$